LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **APRIL 2023**

UMT 5501 – REAL ANALYSIS - II

Date: 29-04-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

Part A

Answer ALL questions **(10 x 2 = 20)**

1. Evaluate: $\lim_{x\to 2}\left(\frac{x^3-7x}{4x^2-5x}\right)$.
2. Define Cluster point, give an example.
3. Define right-hand limit of a function f at a point c.
4. Define Signum function.
5. Using Caratheodory's theorem, find the derivative of $f(x) = x^3$.
6. Give an example to show every continuous function need not be differentiable.
7. Define tagged partition.
8. Give two examples of step function.
9. What is meant by Cantor set?
10. Define open set, give an example.

Part B

Answer any FIVE questions **(5 x 8 = 40)**

11. Let $f: A \to \mathbb{R}$ and if c is a cluster point of A, then prove that f can have only one limit at c.
12. (a) If p is a polynomial function, then prove that $\lim_{x\to c} p(x) = p(c)$.

    (b) Using squeeze theorem of limit, prove that $\lim_{x\to 0}(x)^{\frac{3}{2}} = 0$. **(4+4)**
13. State and prove the location of roots theorem.
14. Let $A \subseteq \mathbb{R}$, let f and g be functions on A to R and let $k \in \mathbb{R}$. If f and g are continuous at c, then prove that $f+g,\ f-g,\ fg$ and kf are also continuous at c.
15. State and prove Bolzano's intermediate value theorem for continuity.
16. State and prove Cauchy mean value theorem.
17. State and prove fundamental theorem of calculus.
18. Prove the following:

    (i) The union of an arbitrary collection of open subsets in $\mathbb{R}$ is open in $\mathbb{R}$.

    (ii) The intersection of any finite collection of open sets in $\mathbb{R}$ is open in $\mathbb{R}$.

Part C

Answer any TWO questions **(2 x 20 = 40)**

19. (a) State and prove sequential criterion theorem for limits.

    (b) State and prove the Maximum-Minimum theorem. **(10+10)**

20. (a) State and prove uniform continuity theorem.

    (b) State and prove Rolle's theorem. **(10+10)**

21. State and prove Taylor's theorem.

22. (a) If $f: [a, b] \to \mathbb{R}$ is continuous on $[a, b]$, then prove that $f \in \mathcal{R}[a, b]$.

    (b) Prove that a subset of $\mathbb{R}$ is closed if and only if it contains all its cluster points. **(10+10)**

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